Short Introduction into
Inertial Measuring Technology

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Acronyms of Functions

Industrial/MIL Projects / Industrie- & MIL-Projekte

CEO	Chief Executive Officer (Geschäftsführer)
CUST	Customer (Kunde)
DE	Design Engineer / Development Engineer (Entwicklungsingenieur)
HD	Head of Development (Entwicklungsleiter)
PGM	Program Manager (Projektmanager)
PJM	Project Manager (Projektleiter)
PM	Production Manager (Fertigungsleiter)
QA	Quality Assurance (Qualitätssicherung)
QM	Quality Manager (Qualitätsmanagementbeauftragter)
SE	Support Engineer
HoS	Head of Support

Aviation & Space Projects / Luft- und Raumfahrtprojekte

AM	Accountable Manager
CUST	Customer (Kunde)
DE	Design Engineer / Development Engineer (Entwicklungsingenieur)
HD	Head of Design (Entwicklungsleiter)
HoA	Head of Office of Airworthiness (Leiter Musterprüfleitstelle)
HoD	Head of Design Organisation
PGM	Program Manager (Projektmanager)
PJM	Project Manager (Projektleiter)
PM	Production Manager (Fertigungsleiter)
CVE	Compliance Verification Engineer (Musterprüfingenieur)
QA	Quality Assurance (Qualitätssicherung)
QM	Quality Manager (Qualitätsmanagementbeauftragter)

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1 SCOPE

This document gives a very short introduction into inertial measuring technology and the understanding of coordinate systems. It will not give a lesson about inertial navigation (for this a lot of excellent literature is available on the market), but it will enable you to understand the most important expressions and the physical background before reading a user manual about an inertial measurement system and using it within your application.

The document also provides some useful information on how to install the physical navigation system as well as the GNSS antenna(s), the odometer, or the magnetometer.

In the following chapters, we briefly cover some basics of inertial measuring technology and related notations. Then information about data processing is given and impacts on several sensor parameters are discussed.

For a more detailed treatment of the topic, we refer to the appropriate literature. In particular, you can find also some useful papers at the download area of iMAR's web site (www.imar-navigation.de).

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2 COORDINATE SYSTEMS AND COORDINATE FRAMES

2.1 Definitions

For simplifying calculations and further analysis, we often represent data with respect to suitable reference systems. On the one hand, we may be interested in determining global or absolute coordinates of given data. On the other hand, coordinate systems fixed on the navigated vehicle are useful for examining local effects. Following, we present important coordinate systems arising from inertial measuring technology.

2.1.1 Body Frame, Vehicle Frame, Navigation Frame and Coordinate Systems

The coordinate frames below are Cartesian, i. e. each is a right-handed trihedron consisting of three orthogonal coordinate axes labeled subsequently as x, y, and z.

Coordinate Frames:

- The "*body frame*" – also known as platform or enclosure frame – is a coordinate system attached to the housing of the inertial measurement system. We usually align this frame with physically existing faces or edges of the device: Often the x/y-plane is parallel to the mounting panel of the internal Inertial Measurement Unit (IMU). In addition, our inertial measurement devices in general follow the convention that the x-axis points from the connectors site to the opposite panel. The origin of this coordinate system is located within the intersection point of all accelerometer axes. The z-axis can be directed up or down, depending on the setup.
- The "*vehicle frame*" denotes the coordinate system tied to the vehicle on which the INS is mounted. The

Figure 1: Body Frame

origin of this frame is the same as for the *Body Frame*, but both frames can be rotated against each another (so-called “axes misalignment”). For the orientation, the Vehicle-x-axis points longitudinally forward (in the major moving direction) and the Vehicle-y-axis faces laterally (right or left). Consequently, the z-axis points vertically (from bottom to top or from top to bottom, depending usally on the application; airborne and shipborne: z downwards; land applications: z upwards).

- The “*navigation frame*” is the locally levelled coordinate system located at the origin of the *vehicle frame*. The x/y-plane is horizontally aligned (with either x or y pointing north) while the z-axis points vertically (down or up, respectively). This fixes only the straight lines of the coordinate systems and leaves some freedom in setting the orientation of the coordinate axes. See chapter 2.1.2 and 2.1.3

Geocentric Coordinate Systems:

- The “*Earth-centerd Earth-fixed coordinate system*” (“ECEF frame”) is a Cartesian spatial reference system that represents locations in the vicinity of the Earth. It is defined with its origin in the center of the Earth.
- The “*geographical coordinate system*” or “terrestrial reference system” is defined by the geoid of the Earth and it is not localy leveled. The height is also different from the one of the “*navigation frame*”. For WGS84 and ETRS89 see chapter 5.

  WGS84 (World Geodetic System) is an ECEF geodetic Cartesian reference frame. In this frame continental plates are drifting.

  ETRS89, as an example, is an ECEF geodetic Cartesian reference frame, in which the Eurasian Plate as a whole is static, i.e. the coordinaes in Europe based on ETRS89 are not subject to change due to continental drift.

For an example of the main axes of a vehicle, see Figure 2 below.

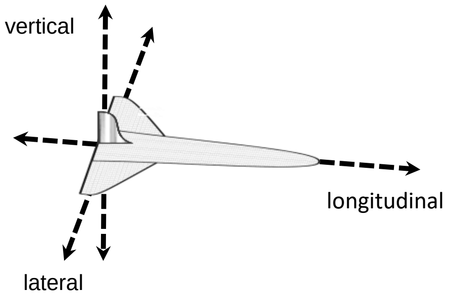

Figure 2: Main vehicle axes; exemplified for an airplane

Since we are usually interested in the values with respect to the vehicle axes, the IMS provides inertial data as well as attitude and heading in the *vehicle frame*. Note that all lever arm coordinates refer to this coordinate system as well. However, the inertial measurement (like acceleration, angles, and angular rates) actually takes place in the *body frame*. For correcting the angular misalignment of the body frame and the vehicle, you must determine their relative orientation (“*misalignment angles*”) accurately.

For defining the *navigation frame*, the following two conventions are commonly in use. Please see the label on the device housing for details about the actual frame of your IMS.

In the following, the arrows depicting the coordinate system axes will point in the direction of positively measured acceleration and velocity as well as the positive direction (of the rotation vectors, right handed) of angular rates.

2.1.2 ENU coordinate systems

The axes of the "East-North-Up" (ENU) Navigation Frame, also called ENU Navigation Coordinate System, are oriented as follows (this frame is in operation usually for land vehicles):

x-axis	directed to *East*,
y-axis	directed to *North*,
z-axis	directed *up*.

It is useful to pick a likewise orientation regarding the z-axis for the vehicle frame of the vehicle:

x-axis	longitudinal in *forward* direction of the vehicle,
y-axis	lateral to the *left*-hand side,
z-axis	upwards to the *top* of the vehicle.

We can preconfigure the body frame of your IMS according to the same orientation. You can recognize this by the housing label shown in Figure 3.

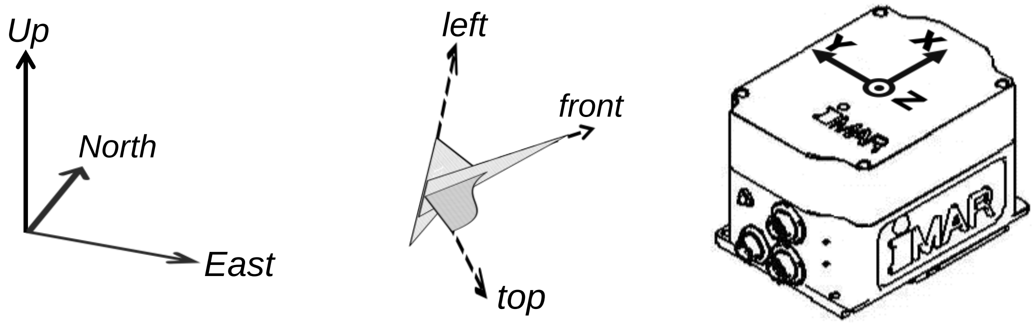

Figure 3: **ENU** Navigation Frame (left), vehicle frame (centered), and example of housing frame (right)

2.1.3 NED coordinate systems

A second variant used, for instance, for aircrafts and watercrafts, is the so-called "*North-East-Down*" (NED) Navigation Frame, also called NED Navigation Coordinate System, which conforms to the directions traditionally used in long range navigation. Respecting the traditional magnetic compass direction and angle, it yields the following navigation frame orientation:

x-axis	directed to *North*,
y-axis	directed to *East*,
z-axis	directed *down*.

A compatible *vehicle frame* convention for the craft results as follows:

x-axis	longitudinal in *forward* direction of the vehicle,
y-axis	lateral to the *right*-hand/starboard side,
z-axis	downwards to the *bottom* of the vehicle.

Again, we can provide your IMS with a *body frame* that respect the same orientation. Please see Figure 4 for the corresponding housing label.

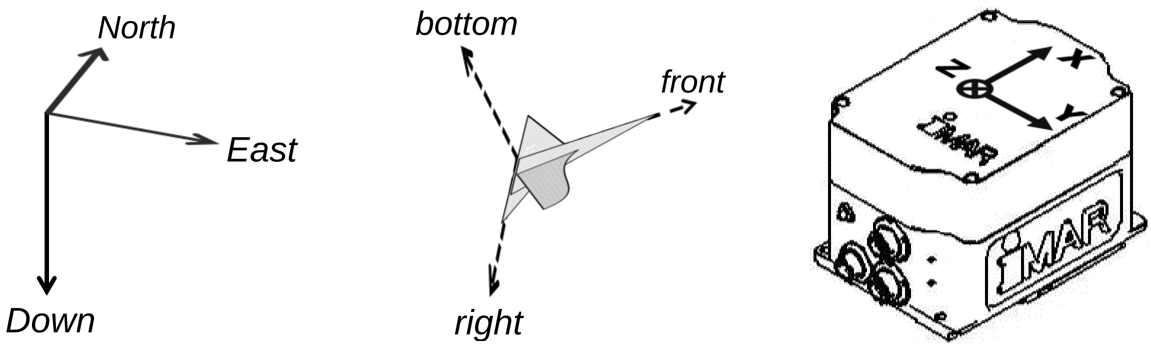

Figure 4: NED Navigation Frame (left), vehicle frame (centered), and example of body frame (right)

2.1.4 True North and Magnetic North, Heading and Yaw, Sign of Height and Altitude

Using this before explained convention of NED, the value of the z-coordinate is negative for positions (altitude / height) over (on top of) sea level. Therefore, altitude – as defined as the height above the reference ellipsoid of the WGS84 system[1] – is defined as the value -z. It is important to note that, in this case, the (vertical) z-component of the velocity is positive (!) when the altitude is decreasing (!).

Also, it must be considered, that the “heading” belongs to a traditional definition: It is common use, that the “true heading” is defined as the clockwise azimuth, starting at geographical north with 0° (attention: the definition of “magnetic heading” is also defined as the clockwise azimuth, but starting at local magnetic north with 0°). The yaw is defined around the z-axis, according to ENU or NED up or down directed.

The next section covers the geometric transformation corresponding to the relative orientation of two coordinate systems like navigation frame and vehicle frame.

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[1] The World Geodetic System 1984

2.2 Eulerian Angles

The Eulerian angles denote rotations for transforming coordinates from an orthogonally trihedral coordinate system (x, y, z) into another one ($\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}$), where both share the same origin. For instance, you may consider the navigation frame and the vehicle frame of your vehicle. Figure 5 shows two differently oriented coordinate systems (x, y, z) and ($\boldsymbol{X}, \boldsymbol{Y}, \boldsymbol{Z}$) with respect to the NED convention. The Z-axis is pointing down (centered). The navigation frame is located on the left and the vehicle frame on the right.

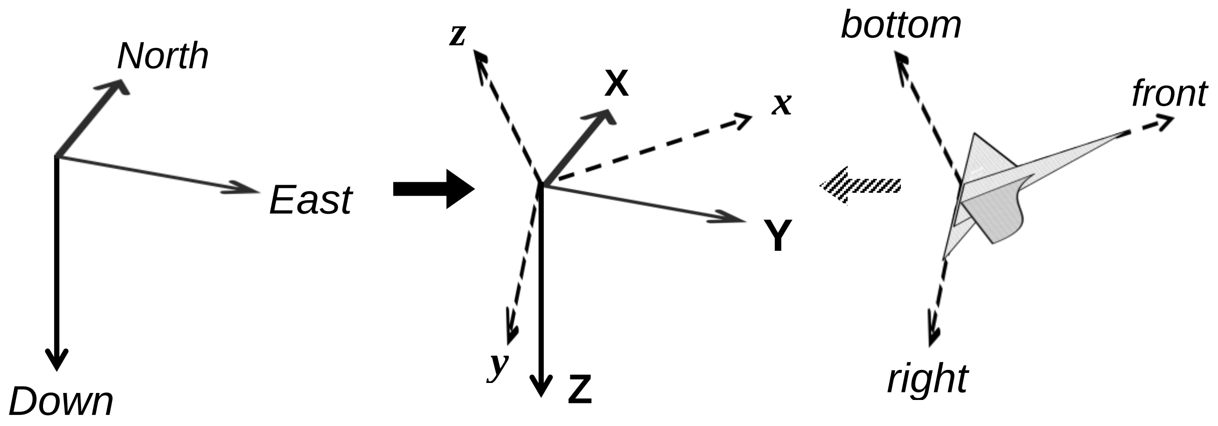

Figure 5: Differently oriented coordinate systems (x, y, z) and (**X**, **Y**, **Z**) with respect to the NED convention

Figure 6 shows two differently oriented coordinate systems (x, y, z) and (**X**, **Y**, **Z**) in the ENU convention setup, with the Z-axis pointing up (centered). Again the navigation frame is located on the left and the vehicle frame on the right.

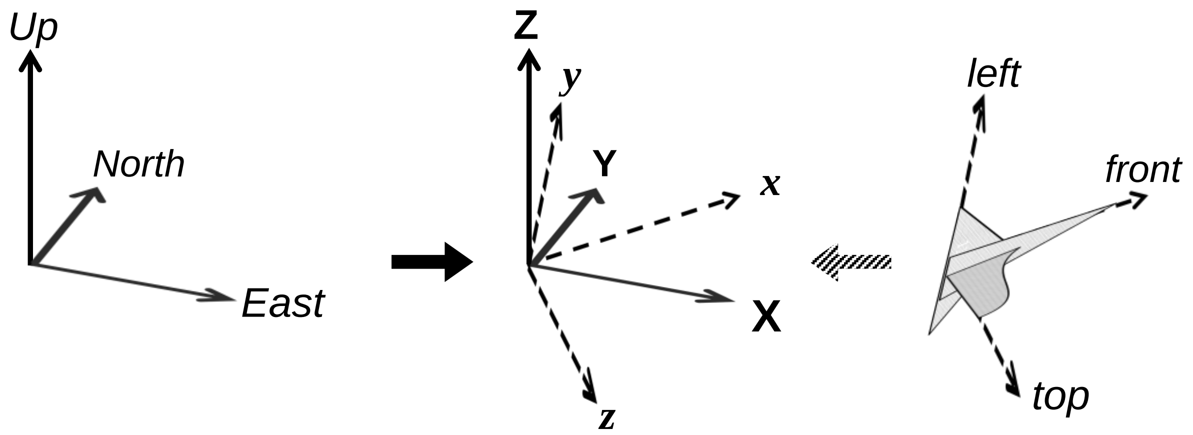

Figure 6: Differently oriented coordinate systems (x, y, z) and (**X**, **Y**, **Z**) with respect to the ENU convention.

Note that the transformation for coordinates **from** the frame (x, y, z) **to** the frame (**X**, **Y**, **Z**) is given by the angles that result from rotating the coordinate axes of the **desired** frame (**X**, **Y**, **Z**) **to** the axes of the **initial** frame (x, y, z).[2]
The latter are called Euler angles yaw Ψ, pitch θ, and roll ϕ and we define them as follows.

1. Rotate yaw Ψ around the **Z**-axis, until the **X**-axis is in coincidence with the projection ***x'*** of the x-axis into the **X**/**Y**-plane → this yields a new frame (***x'***, ***y'***, **Z**); see Figure 7.
2. Rotate pitch θ around the ***y'***-axis to get the ***x'***-axis parallel to the x-axis → we obtain the new frame (x, ***y'***, ***z"***); see Figure 8.
3. Rotate roll ϕ around the x-axis to bring the ***y'***-axis parallel to the y-axis (which automatically rotates the ***z"***-axis into the z-axis) → this finally results in the new frame (x, y, z); see Figure 9.

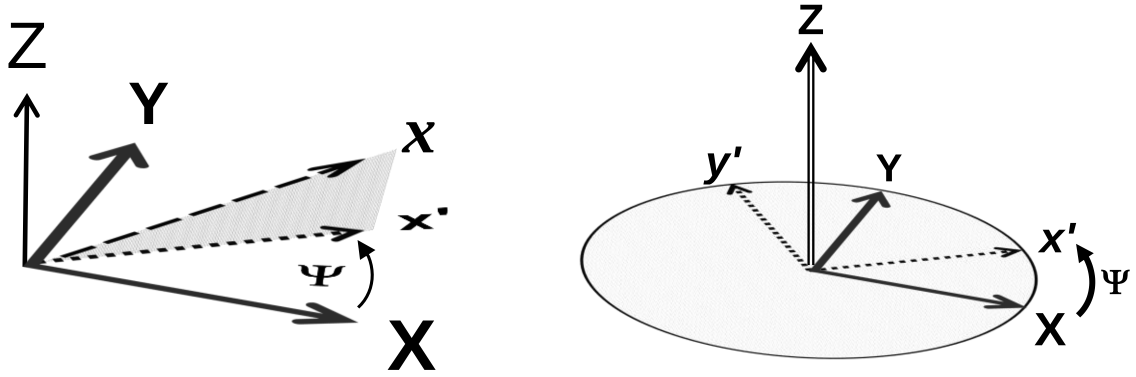

Figure 7: Projection ***x'*** of the x -axis to the **X**/**Y**-plane (left), rotate yaw Ψ around **Z**-axis (right)

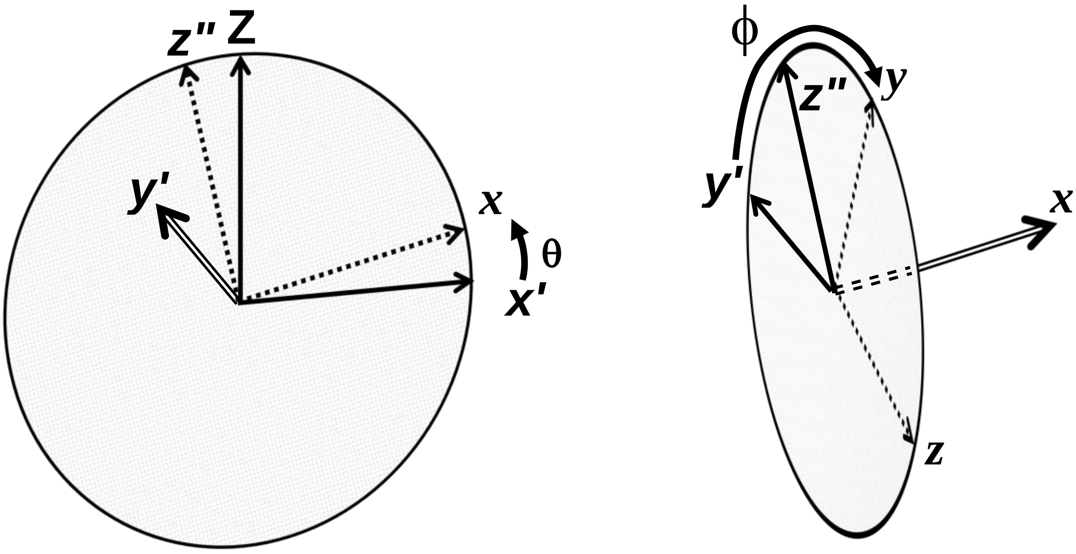

Figure 8: Rotate pitch θ around ***y'***-axis (left), rotate roll ϕ around x-axis (right)

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[2] The angles (computed in the following) deal as angular *offsets*, i.e. rotations that the transformation must undo. Hence, the convention reverses the order of the frames in transformation and rotations.

Figure 9 illustrates the three angles by transforming the **X**/**Y**- to the ***x***/***y***-plane. Since the result depends on the sequential arrangement of the rotations, the order of rotation (yaw → pitch → roll) is essential in defining the coordinate transformation.

The IMS navigation algorithms of course do not utilize the Eulerian angles for their internal calculations; they use them only to input and output data in human readable format. Internally the navigation algorithms (e.g. the strapdown algorithm as well as the extended Kalman filter) use quaternions, because they are free of singularities (Eulerian angles show a singularity at pitch 90° and pitch -90°, in which case the yaw axis and the roll axis are collinear).

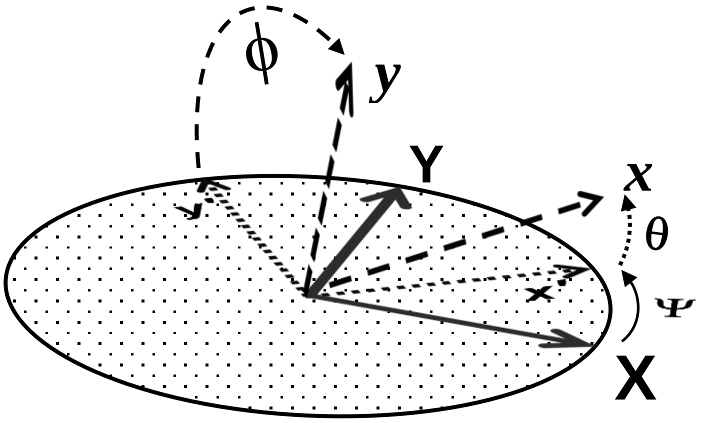

Figure 9: Yaw, pitch, and roll angles (Ψ, θ, ϕ) mapping the **X**/**Y**- to the ***x***/***y***-plane

However, for correcting the angular misalignment of body frame and the vehicle frame, you must determine the relative orientation of these coordinate systems accurately. The resulting three angles ($\Delta\phi$, $\Delta\theta$, $\Delta\Psi$) describe the "*misalignment adjustment values*" or "*bore-sight correction values*", which the IMS software can use for internally transforming all housing frame data into vehicle frame data.

Pitch θ and roll ϕ give the horizontal *alignment*. In contrast to yaw/heading Ψ, the orientation of the velocity vector defines the "*track angle*" ("*course over ground*"). In settings with non-longitudinal movements, slip angles may occur and, hence, yaw Ψ and track angle may be different.

Note that, depending on the navigation frame convention, yaw may differ from the traditional (magnetic*)*[3] *compass angle* as the following section illustrates.

2.3 Yaw Angle vs. Traditional Compass Angle (Heading)

For obtaining the orientation of a vehicle in space, we start with the navigation frame and determine the Eulerian Angles (2.2) for the vehicle frame. According to mathematical convention, an angle increases when rotating around its axis in mathematically positive sense.

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[3] When talking about a „magnetic“ compass, we mean the behavior (approx. 0° corresponds to north), not considering the magnetic deviation etc. Of course, an inertial navigation determines true north while a conventional compass determines magnetic north. Nevertheless, the iNAT systems are able to provide a magnetic heading, derived frm the INS/GNSS based true north and using the World Magnetic Modem (WMM)

Figure 10: Positive rotation around z-axis (left); relationship between yaw Ψ and compass angle $\boldsymbol{\varphi_{North}}$ = heading (right) for ENU coordinates

For the ENU convention, the z-axis of the navigational frame is directed upwards. Figure 10 illustrates the resulting yaw angle Ψ and its relation to the compass angle (heading). In this case, directing the *x*-axis of the vehicle frame to East (i.e. in direction of the first axis of the navigation frame) yields a yaw angle Ψ of 0°. Consequently, the heading towards all four directions of the compass is as follows:

x-axis in East direction:	Ψ	=	0°,	heading	=	90°	
x-axis in North direction:	Ψ	=	90°,	heading	=	0°	
x-axis in West direction:	Ψ	=	180°,	heading	=	-90°	(or 270°)
x-axis in South direction:	Ψ	=	-90°,	heading	=	180°	

In general, we have $\Psi = 90° - \varphi_{North}$ = 90° - heading.
For matching the traditional magnetic compass North angle, the IMS can provide information in NED as well. Like the compass angle, yaw increases clockwise (on top view). This fact results directly from positively rotating round the *z*-axis, which resides upside down by the NED definition; see Figure 11 for details.

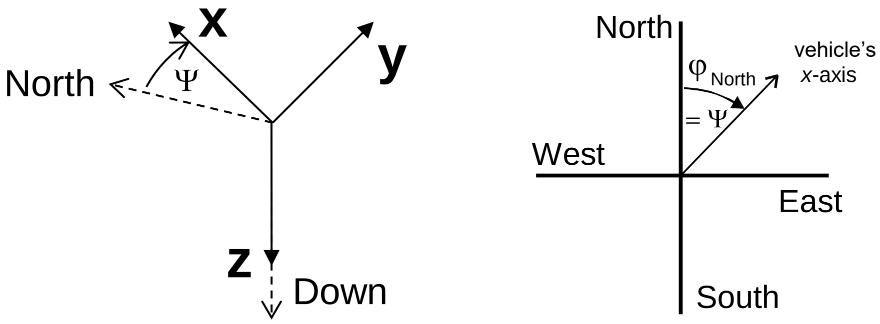

Figure 11: Positive rotation around z-axis (left); relationship yaw Ψ and compass angle $\boldsymbol{\varphi_{North}}$ = heading (right) = yaw for NED coordinates

In this case, yaw angle Ψ and compass angle φ_{North} match, i.e. $\Psi = \varphi_{North}$ = heading and hence, we have:

x-axis in East direction:	Ψ	=	0°,	heading	=	0°	
x-axis in North direction:	Ψ	=	90°,	heading	=	90°	
x-axis in West direction:	Ψ	=	180°,	heading	=	180°	
x-axis in South direction:	Ψ	=	-90°,	heading	=	- 90°	(or 270°)

2.4 Terrestrial Reference System

2.4.1 General Information

If GNSS is used (GPS, GALILEO, GLONASS, BeiDou, …), the GNSS receivers typically provide their data within the Word Geodetic Reference System WGS84, if no position correction data (DGPS, RTK) are provided. WGS84 is the worldwide standard for such data, but it has to be considered that this World Geodetic System is defined by the midpoint of the Earth. As a consequence, a position on the earth shape (e.g. the upper point of the tower of a church or a geodetic surveyed local point) will shows different coordinates in WGS84 if you measure it with a GNSS receiver today and e.g. in ten years.

The reason is that e.g. in central Europe the Tectonic Plate is moving around 2.5 cm per year!

Therefore, geodetic surveyors use specific reference systems, in Europe e.g. ETRS89 (European Terrestrial Reference System). This already contains the continuous plate shift and hence the digital map remains valid over many years (see also chapter 5).

To overcome the WGS84 problem in real time measurements, RTK correction data (augmented GNSS) are typically also transformed to a specific reference system, in Europe often to ETRS89.

Now be careful: If you use such RTK corrections with a standard RTK capable GNSS receiver, you will observe that you achieve good position performance when the RTK correction data are available. However, if the RTK correction data will be lost for a certain duration, the GNSS receiver will switch back to WGS84 and hence you may observe a position jump of several meters!

iMAR has taken action in its systems to avoid such jumps by selecting the appropriate transformation depending on the operational status of the GNSS receiver.

For further understanding:

The Terrestrail Reference System (also called Geodetic Reference System) is defined by several features, which allow to locate points with coordinates:

- Origin of the Terrestrial Coordinate System
- Orientation of its axes
- Scale of its axes
- Definition of its reference ellipsoid

The ITRS (International Terrestrial Reference System) is a prominent realization of a terrestrial reference system. WGS84 (World Geodetic System 1984) is another wellknown Terrestrial Reference System and used for all GNSS measurements.

If uncorrected GNSS data shall be shown on a map it is important to use a map which has its representation also in this reference system. E.g. OpenStreetMap uses WGS84 to make positioning with a pure GNSS receiver most easy. → **Attention:** Therefore, if you use augmented GNSS data (e.g. with ETRS89 corrections), the display on an OpenStreetmap will show significant deviations! So take care about the used data and the used map!

2.4.2 Longitude, Latitude and other Coordinates

Geographic coordinates are given usually in Longitude, Latitude and Altitude:

- The longitude (λ) is the angle in the equatorial plane from the origin meridian to the projection of the point of interest onto the equatorial plane.
- The latitude (ϕ) is the angle in the meridian plane from the equatorial plane to the ellipsoid normal.

Especially in military applications also a local cartesian coordinate frame is used, called Universal Transverse Mercator (UTM, MGRS). Like the traditional method of latitude and longitude, it is a horizontal position representation, which means it ignores altitude and treats the earth surface as a perfect ellipsoid. However, it differs from global latitude/longitude in that it divides the Earth into 60 zones and projects each to the plane as a basis for its coordinates. Specifying a location means specifying the zone and the *x* / *y* coordinate in that plane.

So, a location is described by Zone-Number, Band Letter, Easting Coordinate [m] and Northing Coordinate [m], Altitude [m].

Some applications also use local coordinates. They define a point of origin, an true north related orientation in the horizontal frame and local coordinates (x / y).

2.4.3 Altitude

Several refeences for altitude measurements are used in navigation.

- The **geoid** approximates the Mean Sea Level (MSL) [e.g. EGM2008 (Earth Gravity Model)]. It is s the shape that the ocean surface would take under the influence of the gravity of Earth, including gravitational attraction and Earth's rotation, if other influences such as winds and tides were absent. I.e. all points on a geoid surface have the same geopotential.
- An **Earth ellipsoid** is a mathematical figure approximating the Earth's form, used as a reference frame for computations in geodesy, astronomy, and the geosciences. In geodesy, a **reference ellipsoid** is a mathematically defined surface that approximates the **geoid**, which is the truer, imperfect figure of the Earth – the reference ellipsoid therefore has the same volume as the geoid.

GNSS provides the height above the reference ellipsoid [WGS84]. WGS84 is a global CRS.

Using the undulation correction, the height of the GNSS, measured against the reference ellipsoid in WGS84, can be transformed into the height above the geoid.

The geoid undulation or geoidal height is the height of the geoid relative to a given reference ellipsoid.

Details see chapter 5.

3 PERFORMANCE OF INERTIAL MEASUREMENT SYSTEMS

We distinguish between pure-inertial-north-finding (also called gyro compassing) and non-pure-inertial-north-finding measurement systems.

E.g. the iMAR systems of type iNAT-FSSG-1, iNAT-M200, iNAT-M300, iNAT-U200, … contain gyros of bias day-to-day accuracy of about 0.5 … 100 degrees per hour. By physics, these systems cannot determine true north from measuring the earth rotation rate of 15.04 degrees per hour, but they accurately determine roll and pitch. Furthermore, they can compensate earth rotation rate as well as gravity in the output data. Together with GPS/GNSS aiding and the integrated extended Kalman based sensor data fusion, they also provide true north once they have been operated under sufficient motion and sufficient GNSS constellation.

The systems of class iNAT-FSSG-DA, iNAT-M200-DA, iNAT-M300-DA, iNAT-U200-DA, iATTHEMO etc. provide true north even at standstill by INS/GNSS data fusion using a dual-antenna GNSS receiver. By measuring the relative positional difference between both GNSS antennas, this dual-antenna technology allows to determine the true heading without the need of performing a gyro compassing (and without the need to use high performance inertial sensors for gyro compassing). Under good conditions a dual-antenna GNSS setup can achieve 0.2°/L [m] heading accuracy at standstill condition (standstill is the worst case motion condition) under best GNSS conditions (both antennas to have view to the full sky without multipath), where L is the baseline in meters between both GNSS antennas.

In contrast to the before explained, iMAR's high-performance systems of type iNAT-RQT, iNAT-RQH, iNAT-FSLG-01, iCORUS etc. are true north finding ("gyro compassing") systems with a gyro day-to-day drift of about 0.001 … 0.1 degrees per hour and hence a gyro compassing can be performed without any additional aiding information. I.e. by utilizing earth's rotation rate and gravity for reference and knowing the initial location, these systems can determine true north under motion as well as during standstill. Such systems can also provide "free inertial" navigation without any aiding.

To achieve very high performance even in long duration applications, all systems support aiding by GNSS (i.e. GPS, GLONASS, BEIDOU, GALILEO), optionally including DGPS / RTK / PPP augmentation and odometer aiding. Furthermore true airspeed aiding, magnetometer aiding, doppler velocity log aiding, LiDAR aiding and specific other aiding methods like Zero Velocity Update – ZUPT are available.

3.1 Aiding Information Processing

The INS is performing position, velocity and angles by integrating of acceleration and angular rate. Therefore, the errors of those data are increasing with increasing measurement duration and we talk about a "short time accurate" measurement system. To provide a navigation solution, which is most accurate during short time as well as during long duration, a long-time accurate sensor solution is required, which might even be noisy at short time behavior. GNSS is such sensor with opposite (complementary) error behavior to INS – its noise is quite high compared to an INS solution, but its error is mostly stable independent on time. Other sensors for aiding are velocity sensors like odometer, true airspeed or DVL (Doppler velocity log), magnetometers, LiDAR systems or aiding points like balises or waypoint markers.

To use such aiding information within the Kalman filter based sensor data fusion (e.g. INS/GNSS/ODO), certain constraints have to be fulfilled by the aiding sensors:

- They have to provide a standard deviation of the measurement (e.g. GNSS position or raw data come with a standard deviation obtained in the GNSS receiver) or – if not available – a reasonable standard deviation has to be assumed.
- If they provide velocity, they have to provide a direction in body or vehicle frame (e.g. for odometer) or in navigation frame (for GNSS velocity).
- The lever arm between the aiding sensor and the INS has to be known with sufficient accuracy to allow the data fusion to transform the data of the aiding sensor into the same coordinate system as the INS data.

In the following some basic hints are given how to install the aiding sensors.

3.1.1 Odometer / Wheel Sensor / DVL

Hence, if an odometer is applied as an aiding sensor, it is important that the odometer provides a direction information, otherwise the data fusion within the Kalman filter is not able to perform a reasonable aiding result (in those case, if only the absolute value of the velocity of the odometer is available, the filter would not be able to decide, in which direction the INS would move, especially during slow motions or if the vehicle starts the motion from standstill with very small acceleration).

Therefore, it is mandatory to use a wheel sensor (incremental encoder) with A/B quadrature output (90° phase shifted pulse streams on lines A and B, where the phase shift of +90° or -90° defines the direction, i.e. forward or backward) or with pulses on the A line and direction (high or low level) on the B line.

The lever arm between INS and odometer shall be surveyed in 3D coordinates with a maximum uncertainty of 1/3 of the position accuracy which is expected from the total system.

The iNAT systems also support an automatic surveying mode for the scalefactor, the direction and the lever arm of the odometer during the first GNSS aided motion after power-on.

3.1.2 GNSS Antenna(s)

If GNSS data are used for aiding, it is mandatory to know the lever arm between GNSS antenna(s) and INS. The lever arm between INS and odometer shall be surveyed in 3D coordinates with a maximum uncertainty of 1/3 of the position accuracy which is expected from the total system (if RTK is used with 2 cm position uncertainty, the lever arm shall be surveyed with an uncertainty of less than 5 mm; if standard GNSS is used, the lever arm shall be surveyed with an uncertainty of less than 0.3 m). The navigation system typically supports a procedure to estimate the lever arm during certain vehicle motion.

The GNSS antenna(s) shall be mounted at a location where it has free view to the entire sky to be able to observe as much satellites as possible. The antenna shall be mounted on top of the grounded metal plate (size about 250 x 250 mm or circular with 250 mm in diameter) to reduce distortions from local multipath.

See Figure 12 as an example where two antennas are mounted on our stabilized platform iIPSC-MSG.

The iNAT systems also support an automatic surveying mode for the lever arm of the GNS atenna during the first GNSS aided motion after power-on.

Figure 12: Setup with two GNSS Antennas on Platform iIPSC-MSG

Attention: When installing a dual-antenna setup, it is obligatory that both antennas have the same satellites in view. A configuration in which each antenna covers disjoint parts of the sky only is completely useless, because differences between the observations of the same satellites from both antennas have to be calculated within sub-millimeter range to achieve a reasonable good heading.

3.1.3 Magnetometer

A magnetometer can be used for aiding the INS with magnetic heading information. It has to be mounted in an area of the vehicle without any strong magnetic fields and where the remaining magnetic fields on the vehicle will stay constant during the mission. These constant magnetic fields, generated by hard-iron and soft-iron effects, can be calibrated very well as long as they keep stable. The magnetometer installation does not need a lever arm determination between INS and magnetometer, but it needs a misalignment determination (mainly the difference regarding yaw of magnetometer and INS).

3.2 Processing Algorithms and Applications

The IMS of class iNAT-*xxx* , iATTHEMO, iCORUS, iPST, iTraceRT-MVT, iPRENA, iCOMBANA, iSULONA etc. provide several customizable and pre-installed algorithmic processing options. The main criteria for selecting a suitable algorithm arise from the application requirements and the sensor performance. In this short overview we like to focus on several major application areas:

a) **Vertical Reference Applications:** This includes the area of applications requiring only attitude (roll/pitch), relative heading (change of heading, but no true north), acceleration and angular rate, but no position and no velocity.

b) **Dead-Reckoning Applications:** Measuring a vehicle's track or surveying a road trajectory (road/rail) with high *neighborhood* accuracy requires an odometer to be available. However, it is acceptable to have longer outages of GNSS signals if we do not require highest *absolute* position accuracy (over a long time). Hence, this approach is applicable in subways, basements etc. where we know e.g. an initial heading and where the gyro drift is small enough to keep this heading within a required accuracy over a reasonable time. We assume that the vehicle is moving in forward or backward direction only, but not sideward. This enables us to apply algorithms suited for negligible sideslip angles or to estimate the sideslip angle to correct the result in real-time (e.g. on rail vehicles).

If additionally a good long-time global position performance is required, a continuous aiding with GNSS is necessary, where short outages are allowed (the higher the INS performance, the longer the GNSS outages are acceptable). Using Dead-Reckoning algorithms, the vehicle shall mainly move in the direction of its main axis (forward or backwards, but not significantly sideward, i.e. moving with small sideslip angle).

c) **Aided INS Applications:** Here the device (ship, airplane, road vehicle, UAV, RPV, AUV etc.) is mainly operated under sufficient motion conditions and a sufficient aiding by GNSS or other aiding data is available. The device can move on arbitrary trajectories and the motion is not restricted to forward direction (in this case even motions in all directions and turns improve the navigation performance due to better state observability of the sensor data fusion Kalman filter). The better the INS performance, the longer the duration in which the system can keep high accuracy also during the absence of aiding information (GNSS outages, odometer slippage etc.). High performance systems are able to conzinue in free inertial mode even if all aiding information is lost – the position error increases in this case approximately linear over time (and not quadratically) due to so-called Schuler oscillation.

d) **Unaided INS Applications:** Here no aiding information is available, i.e. the INS has to provide position, velocity, attitude and heading without using any external aiding information. If a true north reference is required, the INS must support gyro compassing.

e) **Special Applications:** These applications require special sensor and algorithm designs, e.g. for transfer alignment applications or horizontal drilling applications or for gimbaled platform stabilization tasks.

The following main algorithms (the design and implementation of several of them is iMAR proprietary) are available, beside of many customized solutions. The algorithms we use for strapdown calculation and data fusion are a result of more than 30 years of experience in using inertial and aiding sensor technology in all areas of applications, from most precise antenna and optronic stabilization on moving carriers to the control of highest speed multi-mach supersonic targets, from deep-sea vessels to ultra-high altitude airborne and space applications, from lightest weight systems to multi-g-resistant systems. For details see: www.imar-navigation.de

I. **ALIGNMENT – Leveling:** This procedure performs an alignment (determining roll/pitch at standstill, i.e. first phase of a static alignment). Leveling (roll/pitch determination) takes place between 2 s and 1 min (depending on environment).

II. **ALIGNMENT – Gyro-Compassing:** This procedure performs an alignment. i.e. determining roll/pitch and also the true heading at standstill or under motion condition. Gyro compassing requires about 3…15 min, depending on gyro angular random walk, gyro bias, dynamic environment and some other impacts.

   a. **Static ALIGNMENT:**

      i. **Systems with gyro compassing capability:** This procedure performs an alignment under static conditions. After a short initial leveling a course alignment will be performed at standstill and after that the fine alignment. This gyro compassing lasts about 3…15 min, depending mainly on gyro angular random walk, gyro bias and required accuracy. If motion is detected during the static alignment, the system switches to dynamic alignment mode automatically.

      ii. **Stored Heading Alignment:** All iNAT systems provide the capability to store the current heading at power-down. If the vehicle is in standstill from the time of power-down until the next power-on, the stored heading can be used to initialize the EKF with the stred heading und to shorten the alignment.

If the system has gyro compassing capability, an automatic “stored heading verification mode” allows to check the validity of the stored heading value before it is used. If the value is not validated to be correct, a standard static alignment is performed.

iii. **Systems without gyro compassing capability:** A static alignment will not provide a benefit and therefore the system performs dynamic alignment. Depending on availability on dual-antenna GNSS or magnetometer the heading can be aided also at standstill or only under motion.

b. **Dynamic ALIGNMENT:**

i. The system will automatically enter this mode as soon as a motion is detected during alignment, otherwise it remains in static alignment mode [4]. The system performs a basic dynamic leveling (taking typically not more than 30 sec) and then a dynamic course alignment is performed under motion conditions. Depending on aiding information availability (odometer, position update by GNSS or by waypoints etc.) the system switches to the dynamic fine alignment and finalizes the alignment.

III. **AIDED INS:** The inertial Navigation algorithm calculates the attitude, heading, position and velocity of the device by using the inertial sensor data (i.e. by calculation of the current attitude and heading from the angular rates and acceleration by integrating the measured acceleration twice over time). The extended Kalman filter algorithm inside the system performs in real-time an INS/GNSS/ODO/xxx sensor data fusion using external aiding information like GNSS aiding, odometer aiding or other aiding information like magnetometer, Doppler Velocity Log, LiDAR information etc. to estimate remaining sensor inaccuracies and to provide an improved solution for position, velocity, attitude, heading and their standard deviations. So-called “*strap down*” algorithms based on coupled non-linear differential equations are used.

During loss of aiding information, by physics the position error initially increases approximately quadratically over time and performs over longer duration an approximately linear increase of the position error due to the typical Schuler oscillation with a period of 84 minutes while navigating within the gravity field of the Earth.

IV. **FREE-INERTIAL INS:** Under this condition the INS is not aided by external sensor data, but certain constraints (like zero velocity updates / ZUPT) can be used to improve the navigation accuracy. If the INS has gyro compassing capabilities, unaided and without ZUPT, the position error of the INS would increase initially quadratically over time and over longer durations approx. linear over time with Schuler oscillation.

V. **INS WITH CONSTRAINTS:** The sensor data fusion of the INS/GNSS/ODO data depends on the application.

- A specific solution is the so-called “*Dead-Reckoning*” navigation mode, where mainly the change of heading together with the measured distance travelled is used to determine a reasonable position. This allows to calculate a relative position in respect to a starting position using odometerand vertical gyro information (simplifed desvription). In this case of non-availability of any position updates (absence of GNSS or waypoint position updates) the navigation error in position increases with the heading error (from gyro drift and scale factor error) and the odometer error (mainly from scale factor error and slippage). This Algorithm is a specific setup of the AIDED INS algorithm.
- Another solution is the determination of attitude and relative heading on a moving object (aircraft, road vehicle) without any aiding information. Special algorithms are used to handle

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[4] Attention: For elder iNAV and iNAT systems, the operator had to preset, whether to perform a static or dynamic alignment. The most recent QNX-based systems do not need this pre-selection. Such devices decide a suitable procedure automatically.

such condition, where backgrund information about certain mission constraints are used inside.

VI. iPEGASUS: Most accurate Transfer Alignment algorithm to allow the transfer of a 3D orientation in space from one location to another location without the need of any aiding information and without the need to know the local latitude (3D Spirit Level).

VII. Other Solutions: Solutions for specific tasks in inertial localization, navigation and surveying, not to be published here (confidential)

3.3 Schuler Oscillation and 24 h Oscillation

"Schuler Oscillation" means a free inertial navigation of an INS after alignment without using any aiding information (no GNSS, no odometer). Only the height (altitude) shall be clamped to a fix value or aided by barometric height to avoid physical caused instability of the free navigating height channel.

3.3.1 Schuler Oscillation on iNAT-RQH-4001 (Example)

The following Figure 13 shows the Schuler oscillation and 24 h oscillation of a high-performance INS (iNAT-RQH-4001)**.** The results are obtained after 20 min alignment and operation under static motion conditions. In real motion environment the results may be different depending on the motion.

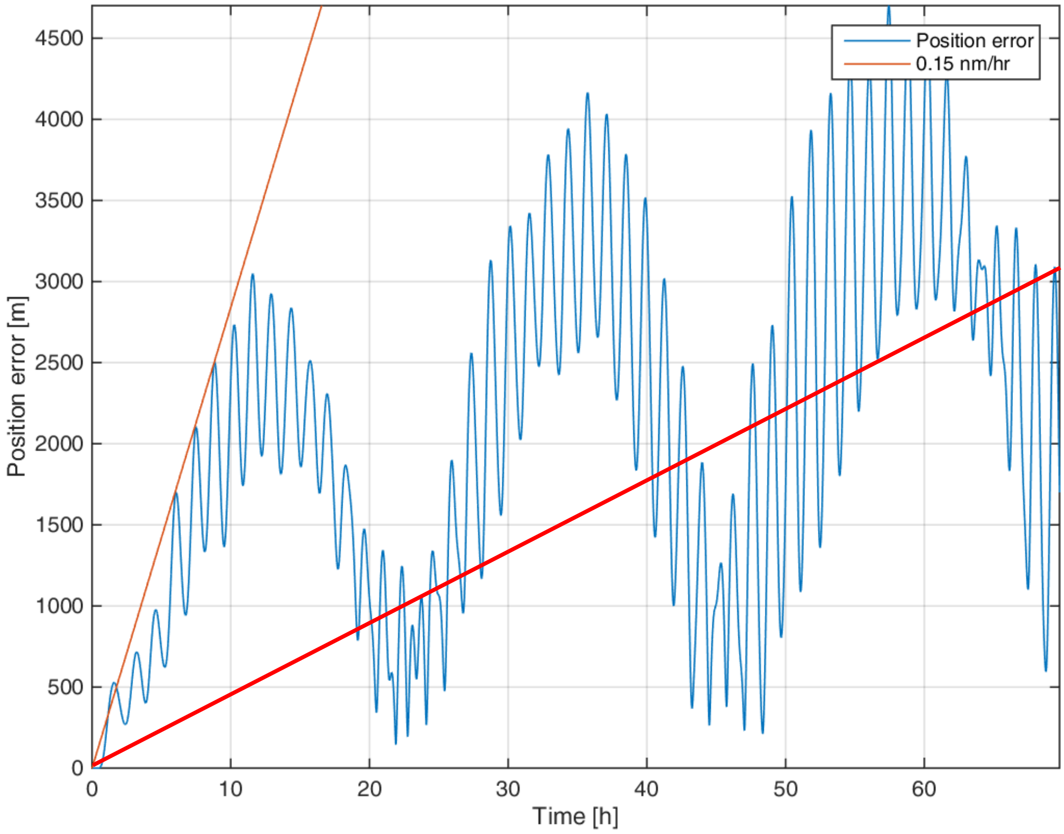

Figure 13: Schuler and 24 h Oscillation (example)

Schuler oscillation: Error bounded navigation on the earth sphere ([nm/h] = nautical mile per hour).

Free inertial error during unaided long-time navigation within the field of gravity:

84 min Schuler period:	0.2 nm/h	(3‘000 m / 10 h)
24 h period:	0.6 nm / 24 h	(3‘000 m / 70 h)

3.3.2 Schuler Oscillation on iNAT-RQT-4003 (Example)

The following Figure 14 shows the Schuler oscillation of a high-performance INS (iNAT-RQT-4003) over 15.5 h**.** The results are obtained after 30 min alignment and operation under dynamic motion conditions over 12 h, where the system was mounted on a 3-axes turntable with excitation of 5° on middle axis (period 17 s, pitch) and 5° on outer axis (period 23 s, yaw). In other motion environment the results may be different depending on the motion.

Schuler oscillation: Error bounded navigation on the earth sphere ([nm/h] = nautical mile per hour).

The plot shows the oscillation (horizontal position over time) over 11 Schuler periods (15.5 h)

Free inertial error during unaided long-time navigation within the field of gravity:

84 min Schuler period:	0.5 nm/h	(specified value: 1.5 nm/h rms = root-mean-square)
After 15 h:	0.33 nm/h	(6‘000 m / 10 h)

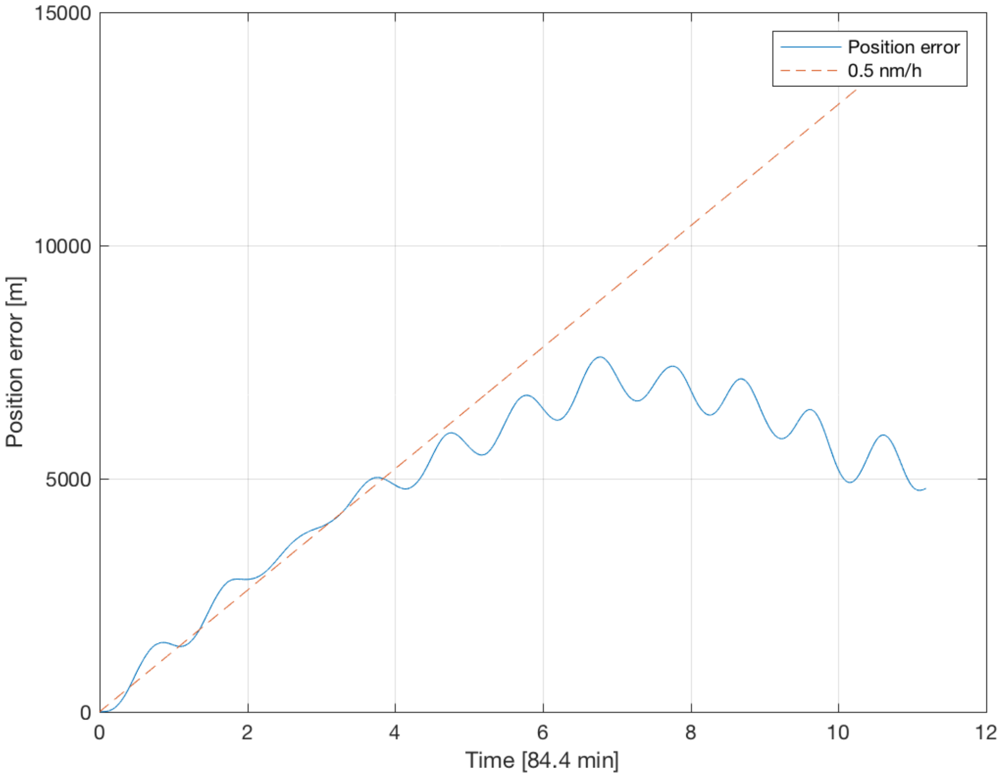

Figure 14: Schuler over 15 h (example)

The next figure shows the “free inertial” navigation error in Longitude / Latitude:

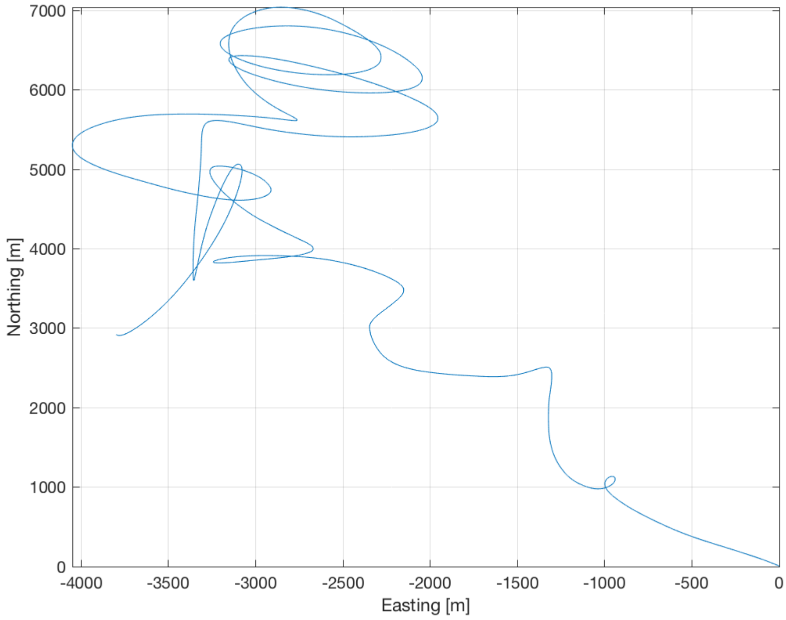

Figure 15: Schuler oscillation as Longitude / Latitude plot (iNAT-RQT-4003)

The following figure shows the regarding velocity value over time.

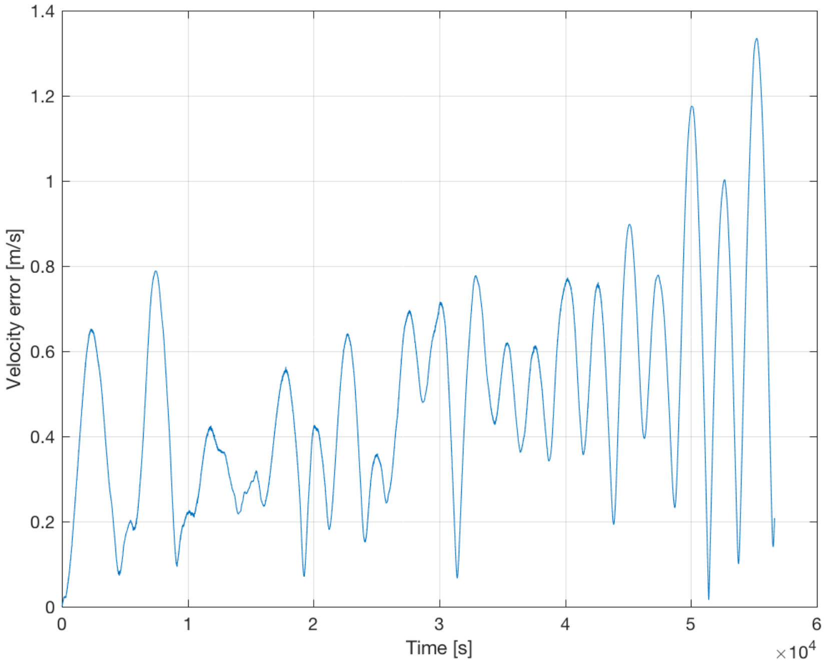

Figure 16: Velocity output during Schuler test

Much lower position drift of a free inertial navigating system (e.g. iNAT-RQT-4002) can be achieved if the system is operated under sufficient dynamics and sufficient GNSS availability for about 24 h before entering the “*free inertial*” mode. Under these conditions a performance of 1 nm / 120 h can be achieved due to the excellent inertial sensor stability.

4 GENERAL INERTIAL SENSORS AND DATA FUSION ASPECTS

Inertial guidance systems were originally developed for navigating rockets, today they are used in many applications from horizontal directional drilling up to space vehicle navigation. Today everybody is daily in touch with inertial sensor technology, for example every modern car contains at least one gyro and two accelerometers for ESP (electronic stability program or more complex advanced driver assistance systems [ADAS]) or for the airbag control to make travelling as safe as possible even in difficult environment. Also, nearly every smartphone contains accelerometers and gyroscopes today.

A typical inertial navigation system uses a combination of roll, pitch and azimuth gyroscopes for stabilizing the x, y and z accelerometers. This solves a large set of differential equations for converting these readings into estimates of velocities, position and attitude, starting off from a known initial position of latitude and longitude.

Today’s implementation of inertial navigation systems (INS) is typically in so-called strap-down technology, where all inertial sensors (gyros and accelerometers) are stiff mounted (strapped down) on the vehicle. In the past the systems had been designed in so-called gimballed technology, where the gyros had been used to stabilise the accelerometers mechanically in space. In strap-down systems the stabilisation is done mathematically, and therefore all inertial sensors suffer the full vehicle’s dynamics. Due to missing mechanical gimbals the strap-down systems are much more robust in operation than the gimballed systems.

All inertial navigation systems suffer from the integration of drift over time, as small errors in measurement are integrated into progressively larger errors in velocity and especially position. This is a problem that is inherent in every open loop control system.

Inertial navigation may also be used to supplement other navigation systems, providing a higher degree of accuracy than is possible with the use of any single navigation system. For example, if, in terrestrial use, the inertially tracked velocity is intermittently updated to zero by stopping, the position will remain precise for a much longer time, a so-called "zero velocity update" (ZUPT).

Control theory in general and Kalman filtering in particular, provide a theoretical framework for combining of the information from various sensors – so-called sensor data fusion. One of the most common complementary sensors used for aiding INS based systems is a satellite navigation system such as GNSS (GPS, GLONASS, GALILEO, BEIDOU).

In the following we have noticed common questions to the integrator / operator of an inertial measurement system:

Dynamical Environment	It is a big difference to operate an inertial measurement system in static or low dynamic environment or in the "real-world": • Check the performance of the IMS (IMS = inertial measurement system) for the environment you want to operate the system in. Will it be used on an aircraft (transportation aircraft, helicopter, drone or fighter?), on a rail vehicle (surface or underground?), a passenger car or a truck or a tank, on a naval ship, a ferry or a speed boat or on an underwater surveying vehicle or inside of a rocket or a torpedo? Or will it be used in a drilling application or in pipeline surveying or in machinery guidance? • Compare the conditions of the data sheet of the system and the conditions in your application:

  - E.g. will GNSS be available in the way as it is assumed for the data in the data sheet of the system?
  - What is the behavior of the system under coning motion? Has the selected IMS sufficient high data rate?
  - Which is e.g. the typical expected motion dynamics of your vehicle?
  - How does the system's parameters influence the desired performance?
  - What operation mode is required (free inertial navigation, aided navigation, surveying, ZUPT operation, control and guidance or something else...) for your mission?
- Take into mind that, also if you only want to know the motion of one single axis (e.g. only roll angle or only heading), under dynamic conditions in general a three-axes measuring system (3 angular rate sensors and 3 accelerometers) is required to achieve the requirements of the application.
  In general, it is not possible to calculate a single axis motion in multi-axes excitation (solution of a non-linear transformation differential equation based on quaternions or direction cosine matrix) with sufficient accuracy using a single axes gyro or using one high accurate gyro and two lower grade gyros. The motion error due to scale factor errors of the inertial sensors is always dominated by the lowest performance gyro installed.

Therefore, it is important that the implemented sensor data fusion (as used in our iNAT systems) is able to estimate also those effects and to calculate the internal sensor data with comparable high data rate, even if only a lower data output rate is required.

- Take into consideration that a MEMS gyro (working on Coriolis law using vibratory excitation) and mechanical gyros (DTG) show a so-called g-dependent drift, i.e. they produce a drift (angular rate offset) dependent on linear or even qadratically acceleration and environmental vibration influence.
  High performance ring laser gyros (RLG = ring laser gyros) and hemispherical resonator gyroscopes (HRG) as well as mid performance fiber optical gyros (FOG) do not show such g-dependent drift.
  It is important to know, that higher performance fiber optical gyros (< 0.1 deg/hr day-to-day bias) suffer significantly from vibration impact and temperature gradients due to physical reasons – typically such fiber optical gyros achieve resonable bias stability only if the temperature gradient is significantly limited, e.g. by power consuming temperature control, for instance to < 0.3 K/min or even less. Have in mind, that typical environmental requirements e.g. on land vehicles are in the range of 2…10 K/min [5].

iMAR uses inside their manufactured systems all state-of-the-art gyro technologies and performance classes from MEMS over FOG and RLG to HRG, dependent from the application requirements and operates a robust, real-time EKF based sensor data fusion with more than 40 states to estimate and compensate most of the residual errors.

Sensor Technology and Data Fusion

Each inertial sensor technology has its specific advantages and drawbacks that you must considere with respect to the desired application and accuracy. Some sensor technologies come e.g. with a very high stability of sensor performance (e.g. ring laser gyros) while others are for instance very light weight and comparable cheap, but affect-

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[5] values are taken from public available product catalogs

	ed by possible aging effects (like MEMS based sensors). Therefore, the signal processing on system level (“sensor data fusion”) has to take care for this. Hence the iMAR data fusion is able e.g. not only to estimate inertial sensor offsets but also to compensate scale factor drifts and other effects in real-time (more than 40 states are estimated, compared to the classical and most common 15 states).
Gyro Bias	If the system operates unaided (without odometer/velocity or GPS or magnetometer aiding), the gyro bias indicates the increase of angular error over time (in °/h or °/s). If the system is aided with speed information (e.g. odometer / wheel sensor or Doppler velocity log), the roll and pitch gyro drift can be compensated in the measurement system and the gyro drift mainly affects the heading accuracy over time. If the system consists of low drift gyros, also the true heading can be estimated using gravity and earth rate information (so-called north-finding). If the system is aided with position information (e.g. by GPS, GLONASS, GALILEO, BEIDOU, LiDAR etc.), also heading drift can be corrected and true heading can be provided (even with medium grade gyros). But of course, the smaller the gyro drift the better are all possible angular corrections and the longer the allowed duration in which the aiding information must not be present (e.g. GNSS in urban canyons)! Also state observability, which is affected by the motion dynamics, is an important subject to system performance. If the system is operated in free navigation mode, the gyro bias is responsible for the position and velocity error over time (so-called *Schuler oscillation*). iMAR’s sensor data fusion contains the capability to estimate inertal and other sensor’s bias.
Gyro Scale Factor Error	This is an indication of the angular error which occurs during rotation. E.g. with 300 ppm Scale factor error (= 0.03 %) the angular error is in the area of 0.1° after one revolution. With a ring laser gyro with 5 ppm or with a high performance fiber optical gyro with 30 ppm scale factor error, the angular error is less than 0.5 arcsec (0.0001’5°) resp. 3 arcsec (0.001°) if the rotation angle is e.g. 30°. iMAR’s sensor data fusion contains the capability to estimate sensor scale factor deviations.
Misalignment	A misalignment between the gyro axes (or accelerometer axes) causes a cross-coupling between the measurement axes. A misalignment of 0.1 mrad inside of the system (e.g. residual calibration mismatch) would lead to a roll error of 0.036° during one revolution around the yaw axis (if the system is unaided). The smaller the required misalignment the higher the requirements to sensor performance and calibration equipment (e.g. iMAR's three-axes turn-tables). iMAR’s sensor data fusion contains the capability to estimate misalignment degradation effects on MEMS based inertial sensors over time.
Accelerometer Offset	An offset on the accelerometer leads to an error during alignment, i.e. determination of initial roll and pitch angle. An offset of 0.1 mg leads to approx. 0.006° angular error (atti-

	tude error). The sensor offsets can be estimated during operation by the system integrated Kalman filter, using GPS or DGPS/RTK data or ZUPT (zero velocity update procedure). iMAR's sensor data fusion contains the capability to estimate sensor's accelerometer bias.
Bandwidth	In general, the dynamic performance of an inertial measurement system is as better as higher the internal sampling rate and the bandwidth of the inertial sensors is. Also, the proper internal data synchronization is very important for accurate signal processing if the IMS is operated under difficult dynamical environment. Therefore, a high precision internal time reference within the IMS is very important. Additionally, a low latency of the data output is mandatory to use an INS for the trajectory or attitude control of autonomous vehicles. All iMAR systems contain a ultra precise clocck reference as well as a dedicated FPGA based sensor data management.
Gyro Random Walk	This value, given in $^\circ/\sqrt{h}$, shows the noise of the used gyro. The higher the noise the more noise is measured on the angular rates and of the angles. Some manufacturers also specify it as the noise density in [°/h/$\sqrt{Hz}$]. Both values are equivalent - if the second value is divided by 60, you get it in [$^\circ/\sqrt{h}$]. An angular random walk of 0.003 $^\circ/\sqrt{h}$ indicates, that the angular error (uncertainty) due to random walk is e.g. 0.000'9° after 6 min (unaided) or 0.000'4° after 1 min (all values one sigma). The angular random walk (ARW) is a stochastic deviation from the nominal angular rate and very important for the accuracy of north finding, because if the random walk decreases (i.e. improves) by times 2, then the needed duration for north finding decreases (shortens) by times 4 (if the resolution of the gyro is high enough).

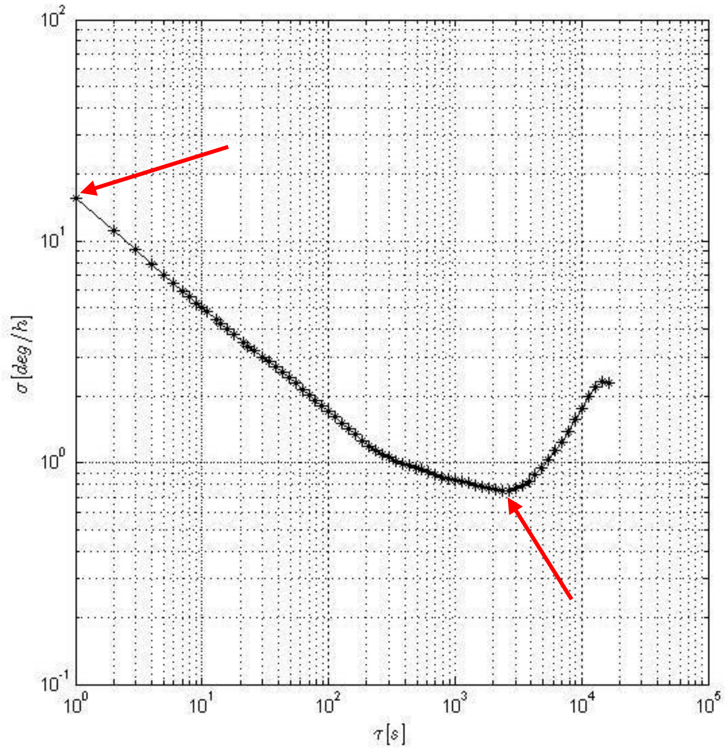

Figure 17: Allan Variance; Graphical representation of the square root of the ARW of a MEMS gyroscope

The plot in Figure 17 of the Allan Variance shows the square root of the ARW of a MEMS gyro graphically (typically take the value at 1 s and divide it by 60 to obtain the ARW in [°/$\sqrt{h}$]).

At 1 s the value of the square-root of the Allan Variance is about 15 °/h. This leads to a value of the ARW of 15/60 °/$\sqrt{h}$ = 0.25 °/$\sqrt{h}$ = 0.0042 °/s/$\sqrt{Hz}$ = 15 °/h/$\sqrt{Hz}$ (white gyro noise assumed). The bias instability (minimum point of the graph) is here 0.8 °/h at a correlation time of 3'000 s. So, it is a quite stable MEMS gyro which is plotted here.

Position error of an *un-aided* INS

We have to distinguish between short-time accuracy and long-time accuracy of an inertial navigation system (INS).

Long-time accuracy of an unaided INS
This value (e.g. given in nm/h, i.e. nautical miles per hour) gives the global position error of an INS due to accelerometer and gyro imperfection, if the system is driven in a so-called Schuler loop operation. Then the position error oscillates with a period duration of approx. 84 minutes as well as with a period of 24 hours. The amplitude of oscillation depends on the accelerometer bias and the "shift" (average of position drift) depends on gyro bias (also called “drift”). The explanation is based on a simplified model assumption; details can be seen from the inertial differential equations).

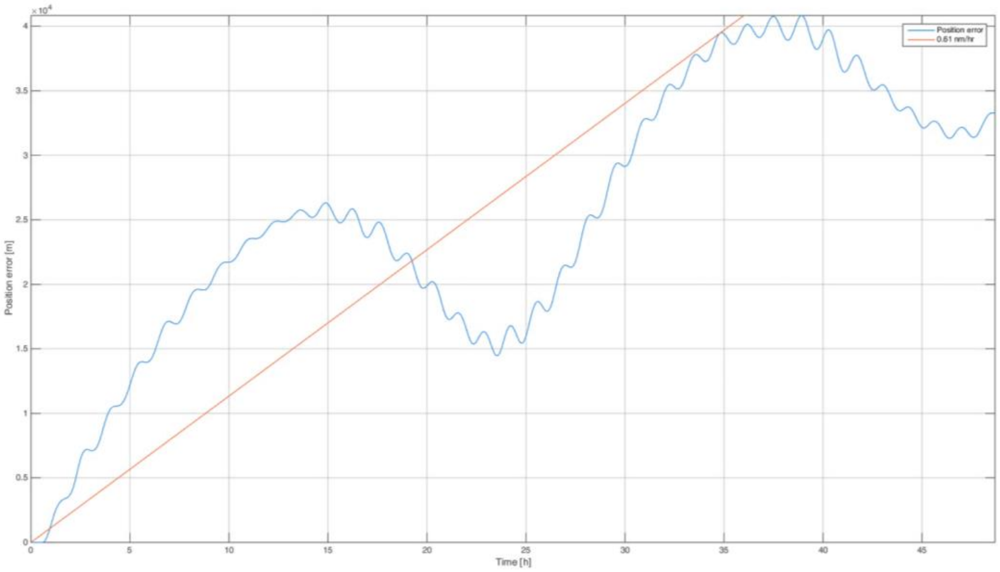

Figure 18: Long-time behavior of a free INS; blue: Position Error; red: approximation with 0.61 nm/hr

The following Figure 18 shows such long-time behavior of a free inertial navigation system (example: data obtained from iNAT-RQT-4003 over more than 2 days):

The position in this plot is given in meters and the time in hours. As an example, the free inertial running INS shows a position error of 34 km after 48 h (i.e. 0.4 nm/h) and during this time the maximum error does not exceed 40 km. The error after 10 h is about 22 km, the drift is about 1.2 nm/h.

To improve the long-time performance of position determination without aiding (no GPS, no odometer!), the system can be set to zero-velocity all x minutes (ZUPT, zero velocity update). During this stand-still period, (example: performing a ZUPT for 5 s all 3 minutes), the Kalman filter is able to estimate the internal sensor errors of the gyros and accelerometers and can improve the position performance dramatically (e.g. position error over 70 km distance with iNAV-RQH-0018 has been shown to be 3 m as an example).

Short-time accuracy of an unaided INS (free inertial navigation)
This value (given in m or m/s) is important for measuring durations less than approx. 20...40 minutes, because Schuler oscillation is not really relevant for short time measurements. An accelerometer offset leads to a position error increasing quadratically over time

$$\Delta s = 0.5 \times \Delta a \times T^2 \; [m] \qquad (1)$$

with Δa = accelerometer offset and T = measuring time.

Example for a medium accurate system:

Δa = 1 mg ≈ 0.01 m/s², T = 100 s → Δs = 50 m

The gyro drift $\Delta\omega$ affects the position error corresponding to the equation

$$\Delta s = \frac{1}{6} g \times \Delta\omega \times \mathrm{T}^3 \; [m] \tag{2}$$

with $\Delta\omega$ in [rad/s] and g = 9.81 m/s².

An attitude (roll/pitch) error of e.g. Δ_attitude affects the position error due to a wrong compensation of the gravity on the horizontal IMS axes:

$$\Delta s = 0.5 \times g \times \sin(\Delta\_attitude\ ) \times T^2 \; [m] \tag{3}$$

Example, how you can validate manufacturer's statements (with data from a vendor's datasheet): IXSEA / iXBlue LANDINS

If someone promotes an IMS with 0.005° roll/pitch accuracy and advertises a horizontal position error of 0.7 m (and a vertical position error of only 0.5 m) after 300 s in free inertial navigation mode (i.e. without odometer aiding, without ZUPT, without GNSS), you can just check and calculate two things with the simple thumb rule equations given above:

Position error result due to 0.005° roll or pitch error after 300 s (free inertial):

0.5 × 9.81 m/s² × sin(0.005°) × (300 s)² = 38 m (from equation (3))

Or looking to the question from the opposite side: What must be the accelerometer accuracy to achieve 0.7 m position error after 300 s (free inertial)?

0.7 m / (0.5 × (300 s)²) = 1.5 µg (!) absolute accuracy (including bias and scale factor impact) over 300 s (from equation (1))

The easy calculation shows the mismatch of the announced performance data (i.e. it is obvious, that the position performance of the related product will be much worse or the attitude error must be much smaller to achieve the advertised performance) – or the given value is not "free inertial".

And for additional information: An absolute accuracy of accelerometer bias of 1.5 µg over 300 sec under real-world environmental conditions is close to gravimeter accuracy but not reliable available in industrial or military land or airborne or shipborne navigation systems. Consider, that already the gravity by itself changes by about 0.3 µg per height meter and the common gravity model by itself does not provide such accuracy! In the lab and under constant environmental conditions, of course, such performance is feasible, but this is not "real world".

So, compare and check information on datasheets carefully before making a decision to use a system (or not to use it).

Position error of an *aided* INS

If the INS is aided, we have to distinguish between position aiding (e.g. by GNSS) and velocity aiding (e.g. by odometer/wheel sensor or GNSS Doppler velocity or Doppler log).

Position aiding

The INS provides high accuracy during short time periods while it shows significant position drift over long-time measurements. GNSS e.g. provides position information with high noise and low data rate, but the position error does not increase over measuring time.

Therefore, using a Kalman filter approach for data fusion, the short-time accurate INS can be coupled with a long time accurate (complementary) position reference system (e.g. GNSS). iMAR's Kalman filter has typically not to be adapted to specific applications, but its architecture allows this easily, if required (e.g. to add additional states for additional constraints, parametrization of covariances, stability analysis etc.). In such applications a solution regarding position will be provided with excellent so-called neighborhood accuracy, but the global position error can never be better than the global position error of the position aiding system (e.g. GNSS). E.g. if GNSS shows a constant position error over a longer time, also the INS/GNSS solution will follow those position error. But using different sources of aiding (GNSS, ZUPT, odometer) the total position error can be minimized.

Velocity aiding / Dead Reckoning

If velocity is provided for aiding (e.g. from a wheel sensor / odometer or from Doppler velocity log) instead of position, the position error of the total Kalman filter solution will grow mainly with the scale factor error of the velocity aiding sensor. If e.g. GNSS aiding is present for a certain time before it will be interrupted (e.g. before the vehicle enters a longer tunnel), the GNSS data will be used together with the IMS and odometer data to estimate the scale factor of the odometer as well as the gyro drift (and in detal also the drift of the heading gyro) precisely and automatically (together with some other installation parameters). This allows also to determine the position of the vehicle during long outages of the GNSS signal with high precision. As an example, using an iNAT-M200/SLN (MEMS based IMS) with wheel sensor, GNSS aiding and integrated data fusion, the position error after 9.7 km GNSS outage is less than 8 m (i.e. < 0.1 %).

Under specific conditions also a mathematical vehicle model can be used for aiding purposes with high performance. Data can be provided on request.

True Heading

The "true heading" performance of an IMS is always an important parameter. If the IMS contains high performance gyroscopes (drift < 0.1 °/h), it can perform an autonomous gyro compassing, i.e. it measures the earth rotation rate, determines the levelling by measuring the gravity vector and calculates from these data the true north (heading) beside of roll, pitch and other values.

If the IMS does not contain such high-performance gyroscopes, it can obtain the true heading only from a combination of a position reference (e.g. GNSS or surveyed waypoints / landmarks) and the inertial sensors, assuming sufficient motion dynamics to be present.

Using only GNSS (without inertial sensors), a so-called "track over ground" can be determined, which is the secant between the obtained GNSS positions over time. Of course, this information shows only the direction of motion of the GNSS antenna over ground, but it says nothing about the heading of the vehicle! Hence with a single GNSS antenna and without additional inertial sensors it is not (!) possible to determine the true heading.

Using a dual antenna system without integrated IMU as stand-alone solution, true heading can be determined as long as both antennas can observe the same (!) GNSS satellites.

If the IMS contains the inertial sensors and a single antenna GNSS receiver (standard setup), using the right signal processing it is easily possible to determine true heading, but this requires that two constraints are fulfilled:

1. The vehicle has to be under translational motion, and
2. The vehicle has to perform certain ("sufficient") changes in heading to provide enough observability to the Kalman filter based sensor data fusion to be able to estimate true heading with sufficient accuracy.

Using a dual-antenna setup together with an IMU (example: iNAT-M300/TLD-DA) the initial heading is determined by the dual-antenna GNSS and the motion of the vehice allows an improvement of the determination of true heading.

Conclusion: An IMS without gyro compassing capability and without dual-antenna GNSS aiding is not able to determine true heading of its carrying vehicle, if the vehicle is moving only on a straight line without changes of direction (this feature is called as "lack of observability"). As soon as a change of heading occurs, the observability is given and the system can provide the desired information. It is very important to take this into account when selecting the right IMS/GNSS solution for the foreseen application. A dual-antenna setup can overcome this issue.

Time Stamping / Synchroni-zation

Especially, if an IMS shall be used for control tasks or for surveying applications, a superior time stamping of the inertial data, odometer data and all other aiding information is mandatory. Therefore, iMAR's measurement systems also provide this feature with very high performance. If a target is moving with 100 m/s, a timing error of 1 ms would already lead to a position error of 10 cm. Consider an RTK aiding with about 1 cm accuracy and you may immediately imagine why a synchronization accuracy with at least 25 µs is mandatory together with a very high internal clock performance.

EMI / EMC Protection

Inertial measurement systems being specifically designed for military or aviation use, contain certain EMI/EMC protection capabilities, but they typically come with quite high weight and cost.

The systems being manufactured by iMAR are designed for the markets of surveying, vehicle testing, aerial laser scanning, pipeline inspection, vehicle and camera stabilization etc. and they are also used within advanced military applications. Due to this wide application area, our systems are mostly protected according to strong standards like MIL-STD461 and MIL-STD704and MIL-STD810 or DO160. This prevents the system from unexpected electro-magnetic interferences and related performance degradation as well as from environmental impacts.

Check the protection level of the system, which you want to apply, against these requirements too. Especially inertial measurement systems being offered for commercial or surveying applications only, sometimes do not provide a sufficient EMI/EMC protection level and this may lead to operational problems in real world's environment.

Open Interfaces

Open interfaces are very important for the user to have highest flexibility in using the system. Interfaces are user-interfaces as well as interfaces to external sensors like GNSS, odometer, depth/altitude sensor etc. The system's architecture should also provide custom specific interfaces if required. For higher volume markets the system shall be designed directly to these applications to meet the economical demands of those applications.

5 APPENDIX: GEOID, ELLIPSOID AND GNSS

This chapter covers the coordinate reference systems (CRS)

Several height models are used in navigation. This chapter gives a short overview. The source of the figures can be found here (status: 22 March 2023):

http://www.esri.com/news/arcuser/0703/geoid1of3.html and https://en.wikipedia.org/wiki/Geoid

The geoid approximates the mean sea level (MSL) [EGM96].

The GNSS provides the height above the reference ellipsoid (h) [WGS84]. WGS84 is a global CRS.

Using the undulation correction, the height of the GNSS, measured against the reference ellipsoid in WGS84, can be transformed into the height above the geoid (H).

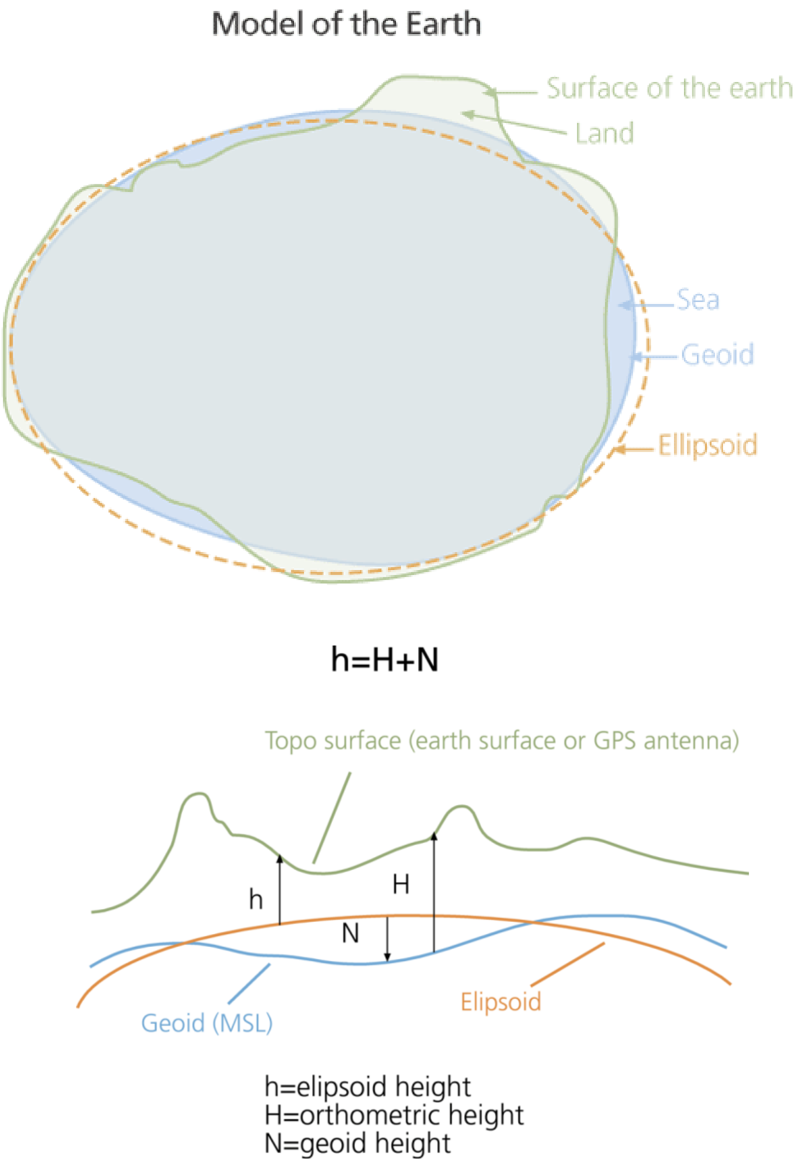

Figure 19: Geoid, Ellipsoid and GNSS Height

Attention: The European Terrestrial Reference System ETRS89 is independent from European continental displacement (approx. 25 mm/year). ETRS89 is a European CRS.